OPTIMIZING AN OSCILLATION ABSORBER IN A COMBUSTION CHAMBER

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We have studied the qualitative behavior of acoustic oscillations in a combustion chamber containing a dynamic absorber. A simple model has been constructed, which describes the forced one-dimensional oscillations. Target functions are proposed, and their behavior is examined in relation to the absorber's control parameters. The absorber may increase the scope for self-excited oscillations. The optimal frictional coefficient in the absorber has been determined for certain target functions. There is an optimal frictional coefficient for each of the proposed functions.

An acute problem in rocket design is to prevent acoustic oscillations in combustion chambers. The characteristic difficulties are that one has inexact information on the main-flow parameters, nonlinearity, and uncertainty over the oscillation source. Consequently, a numerical study of the oscillation equations is undesirable because they are highly approximate. This gives importance to qualitative studies on those oscillations.

Helmholtz resonators are the most effective and common absorbers for acoustic oscillations in chambers or other closed volumes, but in some cases such an absorber can increase the tendency to oscillate, so it is important to choose the optimum parameters of a dynamic absorber correctly. We are not aware of any papers dealing with optimizing dynamic absorbers with allowance for the entire chamber.

Figure 1 shows a typical scheme for using a dynamic absorber, in which 1 is the Helmholtz resonator, while 2 is the amplitude of the pressure oscillations on the fundamental longitudinal mode. There is a sharp turn in the main flow near the nozzle, and there is complete acoustic-wave reflection from it. The oscillations are considered as longitudinal acoustic ones due to oscillation in the rear wall. The oscillations in the resonator are simulated from a mechanical analogy: a load attached to a spring with a viscous damper. The damping behavior for load with spring and direct action on the load has been examined in [1-3]. Detailed studies have been made [2, 3] on a dynamic absorber for a mechanical discrete system (coupled pendulums). In [3], the absorber is considered from the one-dimensional theory for direct action without allowance for the oscillations in the chamber.

Target-function choice is important for absorber optimization. The form of the function is governed by the physical content. We propose functions that enable one to select optimal parameters for dynamic absorbers for longitudinal acoustic oscillations. Numerical tests have been done on these and we have determined the trends in absorber performance as the control parameters are varied. The oscillation amplitude in the chamber and in the absorber may be derived explicitly. Absorber parameters have been determined for which the damping of the free oscillations in the chamber is maximal.

1. Piston with Spring as Absorber: Formulation. Consider a chamber with unit cross-sectional area and length L, at one end of which there is a source of pressure oscillations with a known frequency ω and given amplitude ωA (Fig. 2). A is the maximum deflection of the chamber wall, which bears the source, from the mean position. At the other end of the chamber, these oscillations are damped by an absorber that consists of a piston with mass m on a spring having elasticity k and with frictional coefficient α. Let x be the coordinate axis directed along the chamber and having its origin at one of the ends (Fig. 2), while $\Phi(x, t)$ is the velocity potential of the gas particles in the acoustic wave and Δ the deviation of the piston from the equilibrium position. The wave propagation is described by

$$\Phi_{xx} - \Phi_{tt}/c^2 = 0, \tag{1.1}$$

in which c is the speed of sound in the gas, Φ_{xx} the second derivative with respect to x, and Φ_{tt} the second derivative with respect to time t. The source lies at the rear wall at the chamber, at which the condition for equal velocities is met:

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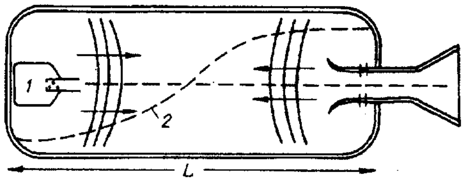

Fig. 1

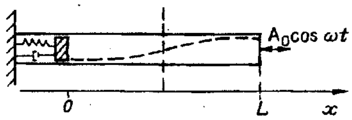

Fig. 2

$$\Phi_x(L, t) = \omega A \cos(\omega t). \tag{1.2}$$

Let the gas have density ρ at rest. The piston's oscillations are described by

$$m\Delta_{tt} + \alpha\Delta_t + k\Delta = \rho\Phi_t. \tag{1.3}$$

Here Δ_{tt} is the acceleration, Δ_t the speed, $\rho\Phi_t$ the force exerted on the piston by the oscillating gas, and $\alpha\Delta_t$ the frictional force acting on the piston. Also, the kinematic condition for equal velocities should be met at the piston:

$$\Delta_t = \Phi_x(0, t). \tag{1.4}$$

We introduce the dimensionless dependent variables

$$\Delta^* = \Delta/A, \quad \Phi^*(x, t) = \Phi(x, t)/\omega AL.$$

Then (1.1)-(1.4) become

$$\begin{gathered}\Phi^*_{xx} - \Phi^*_{tt}/c^2 = 0, \quad \Phi^*_x(L, t) = \cos(\omega t)/L, \\ m\Delta^*_{tt} + \alpha\Delta^*_t + k\Delta^* = L\omega\rho\Phi^*_t(0, t), \quad \Delta^*_t = L\omega\Phi^*_x(0, t).\end{gathered} \tag{1.5}$$

If the oscillations are in the steady state, one can assume that

$$\Phi^*(x, t) = \operatorname{Re}\{[Y\exp(ix\omega/c) + Z\exp(-ix\omega/c)]\exp(i\omega t)\},$$
$$\Delta^* = \operatorname{Re}[X\exp(i\omega t)],$$

in which X, Y, and Z are the complex amplitudes of the corresponding oscillations or waves, X being the piston's oscillation amplitude, Y the amplitude of the wave traveling from the source in the direction opposite to the x axis, and Z the amplitude of the wave reflected from the left-hand end of the chamber and traveling along the x axis (Fig. 2), while i is the imaginary unit. The unknowns X, Y, and Z are derived from (1.5) by solving the inhomogeneous linear system

$$\begin{gathered}Y\exp(i\omega\pi/\omega_c) - Z\exp(-i\omega\pi/\omega_c) = (-i\omega_c)/\omega\pi, \\ (-1 + i\alpha^*/\omega + \omega_p^2/\omega^2)X - i\mu(Y + Z) = 0, \quad X - (\omega\pi/\omega_c)(Y - Z) = 0.\end{gathered} \tag{1.6}$$

For convenience, we introduce additional symbols: let $\omega_p = (k/m)^{1/2}$ be the frequency of the undamped oscillations of the free piston, $\omega_c = c\pi/L$ the fundamental for the acoustic oscillations in the chamber, $M = L\rho$ the mass of gas in the chamber, and μ the ratio of the mass of gas in the chamber to the piston's mass: $\mu = M/m$, $\alpha^* = \alpha/m$.

The fundamental frequency in the piston and chamber is ω_c, the frequency of the natural oscillations in the chamber, so more convenient symbols are

$$\omega = \omega_c(1 + \varkappa), \quad \omega_p = \omega_c(1 + \sigma).$$

Then (1.6) becomes

$$\begin{gathered}Y\exp(i\pi\varkappa) - Z\exp(-i\pi\varkappa) = i/(1 + \varkappa)\pi, \\ wX - i\mu(1 + \varkappa)^2(Y + Z) = 0, \quad X - ((1 + \varkappa)\pi)(Y - Z) = 0.\end{gathered} \tag{1.7}$$

Here and subsequently, $\hat{\alpha} = \alpha^*/\omega_c = \alpha/m\omega_c$; $w = -(1 + \varkappa)^2 + i\hat{\alpha}(1 + \varkappa) + (1 + \sigma)^2$. In these symbols, the solution to (1.7) is

$$\begin{gathered}X = \frac{i\mu(1 + \varkappa)}{\pi w\sin(\pi\varkappa) + \mu(1 + \varkappa)\cos(\pi\varkappa)}, \\ Y = \frac{[\pi w + i\mu(1 + \varkappa)]}{2\pi(1 + \varkappa)[\pi w\sin(\pi\varkappa) + \mu(1 + \varkappa)\cos(\pi\varkappa)]}, \\ Z = \frac{[\pi w - i\mu(1 + \varkappa)]}{2\pi(1 + \varkappa)[\pi w\sin(\pi\varkappa) + \mu(1 + \varkappa)\cos(\pi\varkappa)]}.\end{gathered} \tag{1.8}$$

If the piston's mass is much greater than the mass of gas in the chamber, $\mu = 0$, and the waves are reflected from the piston as from an elastic wall. To simulate real structures, μ should be quite small, since most of a planar longitudinal wave will be reflected from the bottom of the chamber. That reflection is simulated by a large piston mass. Consequently, μ is very small for real structures ($\mu \cong 0$). Let $v = (1 + \sigma)^2 - (1 + \varkappa)^2$, $w = v + i\hat{\alpha}(1 + \varkappa)$; then (1.8) becomes

$$X = \frac{i\mu(1 + \varkappa)}{\pi[v + i\hat{\alpha}(1 + \varkappa)]\sin(\pi\varkappa) + \mu(1 + \varkappa)\cos(\pi\varkappa)},$$

$$Y = \frac{[\pi w + i(\pi\hat{\alpha} + \mu)(1 + \varkappa)]}{2\pi(1 + \varkappa)[\pi w \sin(\pi\varkappa) + \mu(1 + \varkappa)\cos(\pi\varkappa) + i\pi\hat{\alpha}(1 + \varkappa)\sin(\pi\varkappa)]},$$

$$Z = \frac{[\pi w + i(\pi\hat{\alpha} - \mu)(1 + \varkappa)]}{2\pi(1 + \varkappa)[\pi w \sin(\pi\varkappa) + \mu(1 + \varkappa)\cos(\pi\varkappa) + i\pi\hat{\alpha}(1 + \varkappa)\sin(\pi\varkappa)]}.$$

It is convenient to separate the real and imaginary parts of the solution. Let $a = [\pi v \times \sin(\pi\varkappa) + \mu(1 + \varkappa)\cos(\pi\varkappa)]$, $b = \pi\hat{\alpha}(1 + \varkappa)\sin(\pi\varkappa)$, $c = (\pi\hat{\alpha} + \mu)(1 + \varkappa)$, $d = (\pi\hat{\alpha} - \mu)(1 + \varkappa)$, $F = [1/2\pi(1 + \varkappa)]$ then

$$Y = F\frac{[\pi w + ic]}{[a + ib]} = F\frac{[\pi w + ic]\,[a - ib]}{[a + ib]\,[a - ib]} = F\left[\frac{[\pi w a + bc]}{a^2 + b^2} + i\frac{[ac - \pi w b]}{a^2 + b^2}\right],$$

$$Z = F\frac{[\pi w + id]}{[a + ib]} = F\frac{[\pi w + id]\,[a - ib]}{[a + ib]\,[a - ib]} = F\left[\frac{[\pi w a + bd]}{a^2 + b^2} + i\frac{[ad - \pi w b]}{a^2 + b^2}\right].$$

As $ac = a(\pi\hat{\alpha} + \mu)(1 + \varkappa)$, $ad = a(\pi\hat{\alpha} - \mu)(1 + \varkappa)$, $b^2 = \hat{\alpha}^2[\pi(1 + \varkappa)\sin(\pi\varkappa)]^2$, $bc = \pi\hat{\alpha}\sin(\pi\varkappa)(\pi\hat{\alpha} + \mu)(1 + \varkappa)^2$, $bd = \pi\hat{\alpha}\sin(\pi\varkappa)(\pi\hat{\alpha} - \mu)(1 + \varkappa)^2$, it is more convenient to write these expressions as

$$Y = F\left[\frac{\pi w a + \pi\hat{\alpha}\sin(\pi\varkappa)(\pi\hat{\alpha} + \mu)(1 + \varkappa)^2}{a^2 + b^2} + \frac{ia(\pi\hat{\alpha} + \mu)(1 + \varkappa) - \pi^2 v\hat{\alpha}(1 + \varkappa)\sin(\pi\varkappa)}{a^2 + b^2}\right],$$

$$Z = F\left[\frac{\pi w a + \pi\hat{\alpha}\sin(\pi\varkappa)(\pi\hat{\alpha} - \mu)(1 + \varkappa)^2}{a^2 + b^2} + \frac{ia(\pi\hat{\alpha} - \mu)(1 + \varkappa) - \pi^2 v\hat{\alpha}(1 + \varkappa)\sin(\pi\varkappa)}{a^2 + b^2}\right].$$

If $\mu = 0$, then

$$Y = \frac{[\pi w + i\pi\hat{\alpha}(1 + \varkappa)]}{2\pi(1 + \varkappa)[\pi w + i\pi\hat{\alpha}(1 + \varkappa)]\sin(\pi\varkappa)},$$

$$Z = \frac{[\pi w + i\pi\hat{\alpha}(1 + \varkappa)]}{2\pi(1 + \varkappa)[\pi w + i\pi\hat{\alpha}(1 + \varkappa)]\sin(\pi\varkappa)}.$$

Here $\mu = 0$ means that the piston's mass is much greater than the mass of the oscillating gas. The resonant frequencies for the forced oscillations coincide with the natural frequencies of the chamber $\varkappa = 0$.

If the piston's oscillations occur without friction ($\alpha = 0$), the natural oscillations in the chamber will occur at a somewhat different resonant frequency. The oscillation amplitudes are

$$Y = \frac{[\pi w + i\mu(1 + \varkappa)]}{2\pi(1 + \varkappa)[\pi w \sin(\pi\varkappa) + \mu(1 + \varkappa)\cos(\pi\varkappa)]},$$

$$Z = \frac{[\pi w + i\mu(1 + \varkappa)]}{2\pi(1 + \varkappa)[\pi w \sin(\pi\varkappa) + \mu(1 + \varkappa)\cos(\pi\varkappa)]}.$$

Let $\sigma = 0$, which means that the free oscillations of the piston on the spring coincide with the natural oscillations of the gas in the chamber. Resonance effects occur in the chamber for $\omega = \omega_c(1 + \varkappa)$ as the frequency of the driving force if for $\varkappa$ we have

$$\pi[1 - (1 + \varkappa)^2]\sin(\pi\varkappa) + \mu(1 + \varkappa)\cos(\pi\varkappa) = 0.$$

The following approximate expression for the resonant values of $\varkappa_{1,2}$ applies for small $\varkappa$ and μ:

$$\varkappa_{1,2} = \frac{\mu \pm [\mu 2\pi^2(\mu + 4) + \mu^2]^{1/2}}{\pi^2(\mu + 4)} \cong \frac{\pm\sqrt{2\mu}}{2\pi} \cong \pm 0{,}225\sqrt{\mu}.$$

In a real chamber, μ is quite small, so the expression for the resonant $\varkappa$ is sound from the physical viewpoint. The most important point here is that there are two resonant frequencies. This means that a resonator in the chamber may result in an acoustic oscillation because the two closely spaced resonant frequencies for the chamber increase the scope for frequency pulling by hydrodynamic sources [4].

One can optimize the dynamic absorber by varying μ, σ, and α; these three are the control parameters. Parameter $\varkappa$ corresponds to the frequency of the source in the chamber and is freely adjustable. It is necessary to introduce target functions as well as to determine the control and free parameters.

The frequency of the forced oscillations in a chamber in general is controlled by a nonlinear mechanism, so $\varkappa$ should be sufficiently free or independent of the control parameters. Moreover, one can say that $\varkappa$ is chosen for the worst case for certain target functions.

2. Target Functions. The main oscillation sources in a chamber are due to hydrodynamic instability in the main flow, and the acoustic oscillations govern the self-synchronization in the source on account of the hydrodynamic instability or some other nonlinear mechanism [4]. A piston with spring is placed in the chamber to weaken the acoustic feedback, which produces the self-synchronization in the source and thus to lengthen or eliminate the oscillation build-up time.

The target functions are related to the physical and technical content. They may be the acoustic energy, the reflected-wave amplitude, the difference between the acoustic pressures at the opposite walls, the maximum damping coefficient, and so on.

A. Acoustic Energy. The acoustic energy E in the chamber is given by the velocity potential [6, 7]:

$$E(t) = (1/2) \int_0^L \rho_0 \left[|\Phi_x(x, t)|^2 + |\Phi_t(x, t)|^2/(c^2) \right] dx.$$

To calculate E, one needs to represent that potential by means of complex amplitudes. The $|\Phi_x(x, t)|$ and $|\Phi_t(x, t)|$ are calculated from X, Y, and Z. As we have

$$\Delta^* = \Delta/A, \quad \Phi^*(x, t) = \Phi(x, t)/\omega AL, \quad \Delta^* = \mathrm{Re}\,[X \exp(i\omega t)],$$

$$\Phi^*(x, t) = \mathrm{Re}\,\{[Y \exp(ix\omega/c) + Z \exp(-ix\omega/c)] \exp(i\omega t)\},$$

the velocity potential and its derivatives are written as

$$\Phi(x, t) = \omega AL\, \mathrm{Re}\,\{[Y \exp(ix\omega/c) + Z \exp(-ix\omega/c)] \exp(i\omega t)\},$$

$$\Phi_x(x, t) = \omega AL\, \mathrm{Re}\,\{(i\omega/c)\,[Y \exp(ix\omega/c) - Z \exp(-ix\omega/c)] \exp(i\omega t)\},$$

$$\Phi_t(x, t) = \omega AL\, \mathrm{Re}\,\{(i\omega)\,[Y \exp(ix\omega/c) + Z \exp(-ix\omega/c)] \exp(i\omega t)\}.$$

It is convenient to consider the oscillation amplitude as a function of time at a fixed instant t_* instead of $\Phi(x, t)$ or its derivatives, that time being such that $\exp(i\omega t_*) = 1$. Then $\Phi(x) = \Phi(x, t_*)$, $\Phi_x(x) = \Phi_x(x, t_*)$, $\Phi_t(x) = \Phi_t(x, t_*)$. The acoustic energy is

$$E(\alpha, \mu, \sigma, \varkappa, t_*) = (1/2) \int_0^L \rho_0 \left[|\Phi_x(x)|^2 + |\Phi_t(x)|^2/(c^2) \right] dx. \tag{2.1}$$

To calculate E as a function of the control parameters, we substitute the (1.8) amplitudes into (2.1).

The absorber is optimized by choosing α, μ, and σ such that the acoustic energy in the oscillations $E(\alpha, \mu, \sigma, \varkappa, t_*)$ is least for a certain set of $\varkappa$ values. Equally important for applications is the dependence of the energy on the control parameters. A numerical study was made on an example to elucidate the mechanical meaning of the resonant absorber parameter interaction. We examined the acoustic energy in relation to forced oscillation frequency, which is described by the dimensionless parameter $\varkappa$, and the absorption coefficient $\hat{\alpha}$.

Let the ratio of the gas mass to the piston mass remain constant and let the frequency of the free oscillations in the resonant absorber ω_p coincide with the frequency of the acoustic oscillations in the chamber $\omega_c = \omega_p = \omega_c(1 + \sigma)$, so $\sigma = 0$. For convenience we can assume that $L = \pi$ and $A = 1$. In real cases, the frequency of the forced oscillations in the chamber differs little from the natural frequency, so one can assume that $|\varkappa| \ll 1$ and also that μ is quite small, since in a real structure much of the acoustic energy is reflected from the front wall (Fig. 1). For example, one can take $\mu = 0.01$. As μ and $\varkappa$ are

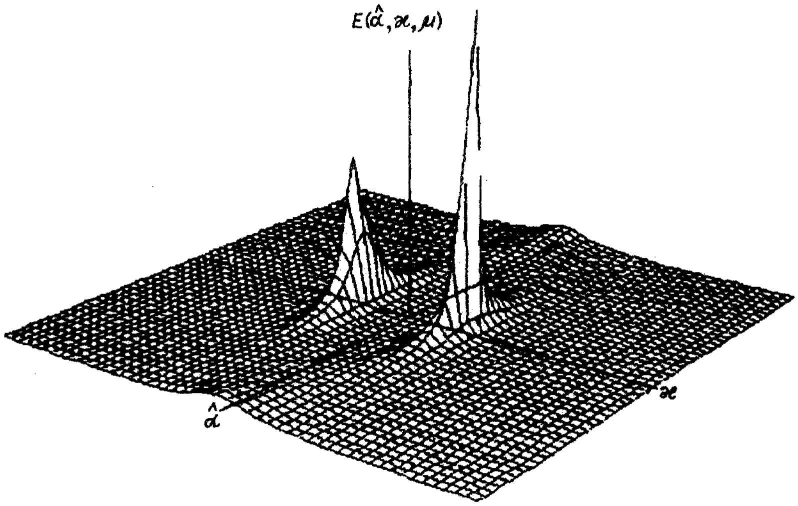

Fig. 3

quite small, the approximate expressions for a and b apply, which are derived by Taylor-series expansion of the corresponding quantities up to the order of $\varkappa^2$:

$$a = [\pi v \sin(\pi\varkappa) + \mu(1+\varkappa)\cos(\pi\varkappa)] = \mu + \mu\varkappa - \pi^2(\mu+4)\varkappa^2/2,$$
$$b = \pi\hat{\alpha}(1+\varkappa)\sin(\pi\varkappa) = \pi^2\hat{\alpha}\varkappa(1+\varkappa), \quad v = -\varkappa - \varkappa^2.$$

Then Y and Z are given by

$$Y = F\left[\frac{\pi w a + \hat{\alpha}\varkappa\pi^2(\pi\hat{\alpha}+\mu)(1+\varkappa)^2 + ia(\pi\hat{\alpha}+\mu)(1+\varkappa) - \pi^2 v\hat{\alpha}(1+\varkappa)\pi\varkappa}{[\mu+\mu\varkappa-\pi^2(\mu+4)\varkappa^2/2]^2 + [\pi^2\hat{\alpha}\varkappa(1+\varkappa)]^2}\right],$$

$$Z = F\left[\frac{\pi w a + \alpha\varkappa\pi^2(\pi\hat{\alpha}-\mu)(1+\varkappa)^2 + ia(\pi\hat{\alpha}-\mu)(1+\varkappa) - \pi^2 v\hat{\alpha}(1+\varkappa)\pi\varkappa}{[\mu+\mu\varkappa-\pi^2(\mu+4)\varkappa^2/2]^2 + [\pi^2\hat{\alpha}\varkappa(1+\varkappa)]^2}\right].$$

We use these to examine the acoustic energy numerically for small $\hat{\alpha}$ and $\varkappa$, the results being given in Figs. 3 and 4 for $\mu = 0.01$ with $\hat{\alpha}$ and $\varkappa$ in the ranges ±0.1 and ±0.2 correspondingly. The most important points are that: a) the two closely spaced resonant frequencies occur for the forced oscillations, b) there is effective suppression for the oscillations as α increases from zero (no absorption) to the optimum value, and c) as α increases further, the acoustic energy increases, and d) there is an optimal absorption coefficient.

B. Reflected-Wave Amplitude. Here one selects the absorber's parameters such that $|Z|$, the amplitude of the wave reflected from the chamber wall opposite the source, is minimal, while Z is the complex amplitude of the wave reflected from the absorber:

$$Z = \frac{[\pi w + i(\pi\hat{\alpha} - \mu)(1+\varkappa)]}{2\pi(1+\varkappa)[\pi w\sin(\pi\varkappa) + \mu(1+\varkappa)\cos(\pi\varkappa) + i\pi\hat{\alpha}(1+\varkappa)\sin(\pi\varkappa)]}.$$

For this target function, the resonant frequencies of the forced oscillations are the same as those for the acoustic energy in the chamber. For small $\varkappa$ and α with $\mu = 0.01$, we examined the reflected-wave amplitude numerically. Figure 5 shows results for $|\varkappa| < 0.1$ and $|\alpha| < 0.1$. Identical qualitative behavior occurs for the energy and the reflected amplitude for small control-parameter values. The conclusions are as for the previous target function.

C. Pressure Difference at Opposite Walls. Here one chooses the control parameters such that the pressure difference at the opposite walls will be minimal. The complex pressure force P(0) on the left-hand wall x = 0 (Fig. 1) on the spring side is represented as

$$P(0) = -(i\alpha\omega_* + k)AX.$$

The pressure force on the front wall P(0) is composed of the frictional force and the spring deformation force. The pressure force P(L) on the real wall is equal to the pressure force from the acoustic wave:

$$P(L) = -i\rho\omega_*\Phi(L) = i\omega_*^2\rho AL(Y+Z).$$

The complex oscillation amplitude is equal to the pressure difference $\Delta P = P(L) - P(0)$:

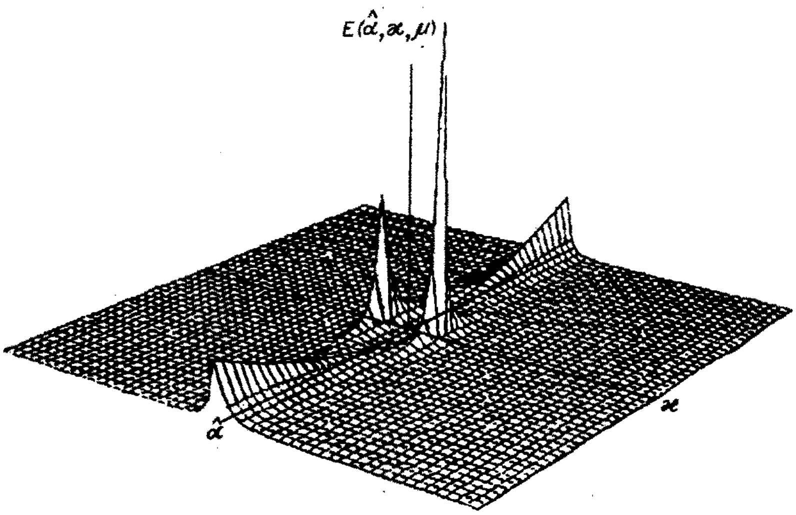

Fig. 4

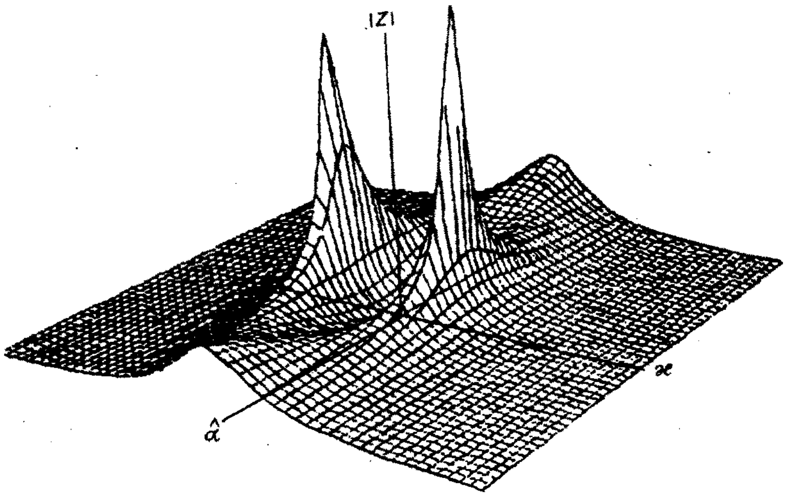

Fig. 5

$$\Delta P = P(L) - P(0) = (i\alpha\omega_* + k)\, AX + i\omega_*^2 \rho AL\, (Y + Z).$$

The absolute value for the oscillation amplitude in the pressure difference is

$$|\Delta P| = |(i\alpha\omega_* + k)\, AX + i\omega_*^2 \rho AL\, (Y + Z)|.$$

This formula has been examined numerically (Fig. 6) from (1.8) for $\sigma = 0$, $\mu = 0.01$, and small values of $\varkappa$ and $\hat{\alpha}$ ($|\varkappa| < 0.1$, $|\hat{\alpha}| < 0.1$). As for the previous target functions, the pressure difference has two resonant peaks for the two values of the resonant frequency. There is no qualitative difference from the previous target functions.

3. Maximal Damping Coefficient. Another way of absorber optimization is to choose the control parameter such that the damping over time in (1.1)-(1.4) (without the oscillation source, A = 0) is maximal. Here it is assumed that oscillations arise in the chamber close to the longitudinal mode in the natural oscillations, and one examines their behavior over time. The oscillations in the resonant chamber—dynamic absorber system are described by (1.1)-(1.4) for the unknown functions $\Phi(x, t)$ and $\Delta(t)$. Here we have incorporated the following: 1) the equation for acoustic-wave propagation in the chamber, 2) the condition for no flow at the right-hand wall of the chamber x = L, 3) Newton's law for the piston, and 4) the condition of no flow at the piston, namely equal velocities for the piston and gas particles:

$$\Phi_{xx} - \Phi_{tt}/c^2 = 0, \quad \Phi_x(L, t) = 0,$$
$$m\Delta_{tt} + \alpha\Delta_t + k\Delta = \rho\Phi_t(0, t), \quad \Delta_t = \Phi_x(0, t). \tag{3.1}$$

Let the oscillations occur with a frequency close to the natural frequency of the oscillations in the chamber and with a certain small damping. Then the oscillations are described by a

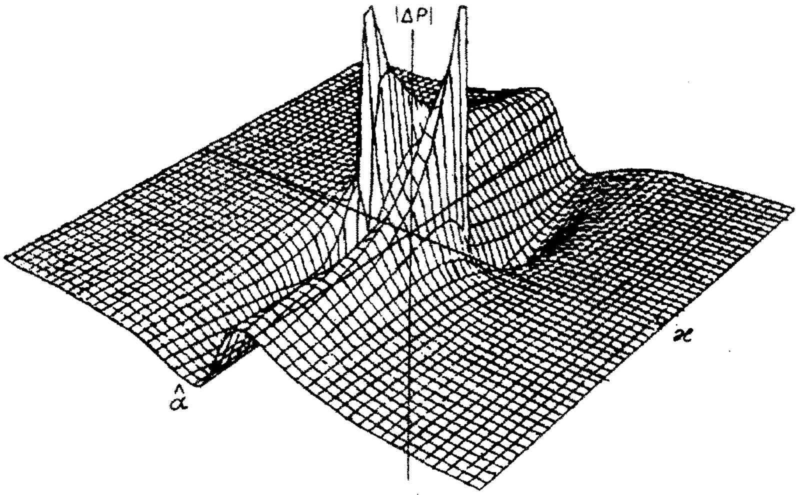

Fig. 6

complex frequency, whose imaginary part describes the damping over time. Absorber parameter optimization here is such as to produce maximum damping. Let $\Phi(x, t)$ and $\Delta(t)$ describing the oscillations of the gas and piston take the form

$$\begin{gathered}\Phi(x, t) = \mathrm{Re}\,[\Phi(x)\exp(i\omega t)] = \exp(-\mathrm{Im}\,\omega t)\,\mathrm{Re}\,[\Phi(x)\exp(i\,\mathrm{Re}\,\omega t)],\\ \Delta(t) = \mathrm{Re}\,[\Delta\exp(i\omega t)] = \exp(-\mathrm{Im}\,\omega t)\,\mathrm{Re}\,[\Delta\exp(i\,\mathrm{Re}\,\omega t)].\end{gathered} \tag{3.2}$$

The imaginary part of ω describes the damping rate in the chamber. It is convenient to assume that ω differs little from ω_c ($\omega = \omega_c + \delta\omega$), with $\delta\omega$ taken as quite small ($\delta\omega = \delta_1 + i\delta_2$). Equation (3.2) enables us to write (3.1) as

$$\begin{gathered}\Phi_{xx} - \Phi\omega^2/c^2 = 0, \quad \Phi_x(L) = 0,\\ \omega^2 m\Delta + i\omega\alpha\Delta + k\Delta = i\omega\rho\Phi(0), \quad i\omega\Delta = \Phi_x(0).\end{gathered} \tag{3.3}$$

Absorber optimization involves selecting the control parameters in (3.3) to be such that δ_2 is as large as possible. Let

$$\Delta = X, \quad \Phi(x) = Y\exp(i\omega L/c) + Z\exp(-i\omega L/c).$$

By virtue of (3.3), X, Y, and Z satisfy

$$\begin{gathered}Y\exp(i\omega L/c) - Z\exp(-i\omega L/c) = 0,\\ (-\omega^2 m + i\omega\alpha + k)X - i\omega\rho(Y + Z) = 0, \quad X - (Y - Z)/c = 0.\end{gathered} \tag{3.4}$$

This system has nontrivial solutions only for those ω (complex eigenvalues) for which the determinant of this system is 0. The complex eigenvalues are the solutions to

$$\det(M) = -i\omega\rho c\,[1 + \exp(2i\omega L/c)] + (-m\omega^2 + i\omega\alpha + k)\,[1 - \exp(2i\omega L/c)] = 0,$$

which is equivalent to

$$\omega c/[\omega^2 - \omega_p^2 - i\omega(\alpha/m)] = (m/\rho)\,\mathrm{tg}\,(\omega L/c). \tag{3.5}$$

For zero friction ($\alpha = 0$), the (3.5) solutions may be derived graphically. The friction is assumed quite small. The representation $\omega = \omega_c + \delta\omega$ enables one to linearize (3.5) with respect to the small quantity $\delta\omega$. The solution to the linearized equation is

$$\delta\omega = c^2(\rho/m)\,\omega_c/2L\,[(\omega_c^2 - \omega_p^2) - i(\alpha/m)\,\omega_c].$$

The real and imaginary components of the (3.5) solution, linearized with respect to $\delta\omega$, are

$$\begin{gathered}\delta_1 = \mathrm{Re}\,(\delta\omega) = c^2(\rho/m)\,\omega_c(\omega_c^2 - \omega_p^2)/2L\,[(\omega_c^2 - \omega_p^2)^2 + (\alpha/m)^2\omega_c^2],\\ \delta_2 = \mathrm{Im}\,(\delta\omega) = c^2(\rho/m)\,\omega_c^2(\alpha/m)/2L\,[(\omega_c^2 - \omega_p^2)^2 + (\alpha/m)^2\omega_c^2].\end{gathered}$$

The largest damping occurs when the friction in the absorber is

$$\alpha = m(\omega_c^2 - \omega_p^2)^2/\omega_c^2.$$

Optimal absorber friction has been confirmed by experiment [8], and methods of increasing it have been described.

4. Conclusions. 1. A dynamic absorber is effective in producing coincidence between the natural frequencies and a certain optimum value for the coefficient of friction. 2. For small coefficients of friction, the dynamic absorber may give rise to oscillations because the chamber with absorber has two similar resonant frequencies, and this extends the scope for oscillation frequency pulling. 3. With large coefficients of friction, the absorber in the chamber may be ineffective.

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